Eur. Phys. J. D **46**, 431–435 (2008)
DOI: 10.1140/epjd/e2007-00335-x

THE EUROPEAN PHYSICAL JOURNAL D

Towards a test of string theory using Rydberg atoms

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Received 29 March 2007 / Received in final form 1 October 2007
Published online 21 December 2007 –

Abstract. The current controversy over the need for an experimental test of String Theory is considered. We report recent experiments on quasi-bound electrons in crossed electric and magnetic fields, in which states of very large electric dipole moment are excited. The excited electron is confined to one side of the atomic nucleus in the outer well of a controllable double-well potential. These states are discussed in relation to a recent theoretical proposal to test the spatial non-commutativity underpinning String Theory by studying Penning orbits of Rydberg atoms in crossed electric and magnetic fields.

PACS. 11.25.-w Strings and branes – 31.30.-i Corrections to electronic structure

1 Introduction

Currently, there is great controversy [1–3] over the usefulness or otherwise of developing String Theory. Although this theory presents an attractive picture of the nature of elementary particles, its predictions escape measurement because of the vanishingly small scale sizes involved (around 10^{-35} m), with the consequence that String Theory, despite some desirable features, is deemed 'unfalsifiable'. If true, such criticism [1–3] would indeed be severe, and might lead one to abandon the attempt. The purpose of the present letter is to argue that this conclusion, in fact, is quite premature, and that measurable effects are predicted by String Theory on normal quantum scales, which the current criticisms have apparently overlooked. Our argument hinges on recent theoretical papers by Zhang [4–9] who has explored new features of deformed Heisenberg-Weyl algebras in non commutative space, with particular emphasis on viable experimental tests. He shows that, with both position-position and momentum-momentum non-commuting, nontrivial dynamics are recovered for vanishing kinetic energy and diminishing magnetic field, so that the lowest angular momentum turns out to be $\hbar/4$. This result provides a clear signal of spatial non-commutativity, which survives into the quantum scale, because the vanishingly small constants usually present in predictions derived from String Theory actually cancel out in the potential he specifies.

Zhang's argument is based on the following set of commutation rules, applied to the Hamiltonian of a Rydberg atom in the presence of crossed electric and magnetic fields:

$$[\hat{x}_i, \hat{x}_j] = i\xi^2\theta\epsilon_{ij} \qquad [\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij}$$
$$[\hat{p}_i, \hat{p}_j] = i\xi^2\eta\epsilon_{ij} \qquad (i, j = 1, 2) \tag{1}$$

where θ and η are constants, independent of position and momentum, η is the antisymmetric unit tensor, and ξ is a constant scaling factor. These relations are quite general in the presence of non-commuting space operators. We refer the reader to Zhang's paper [5] for the conditions which must apply to the potential of the system and information on how the rules should be used. The essence of the matter is that the magnetic field breaks the symmetry between the orderings of the position operators, with the system held on a loose orbit and no charge at its centre. In order to achieve a real experimental test, Zhang proposes to perform experiments on the 'quasi-Penning' orbits of cold Rydberg atoms. His proposal, however, turns out to be unrealistic in practice, for the following reasons. First, the experimental arrangement requires the production of a radial electric field with the atom at its centre, which is a very difficult geometry to achieve. Second, the experiment would require detection at an extremely high characteristic frequency [9], estimated as 10^{22} Hz, which makes it very difficult to conceive of a viable experimental method for Rydberg atoms. In the present article, we argue that Zhang's mathematical approach can be transposed to a very different physical situation, to which his argument equally applies. Instead of measuring the orbits

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of cold Rydberg atoms placed in a suitable combination of external fields, we propose that a more suitable test is to study the orbits of quasi-free electrons placed in a suitable combination of fields internal and external to the atom. The Hamiltonian for the system we propose is very similar to the one considered by Zhang [4–9], and therefore the mathematical structure of his proof can be preserved.

2 Theoretical background

We begin by remarking that the principles on which Zhang's result is based are fundamental to the structure of non-commutative space in String Theory, and must therefore (if correct) apply to all systems or particles, be they Rydberg atoms or electrons. We will replace the atom of mass M by an electron of mass m and also change the field geometry, while seeking to preserve the mathematical structure of the problem he describes.

Our argument is best understood by considering the Hamiltonian of Zhang's system. He proposes an experiment on 'quasi-Penning' orbits of a cold Rydberg atom in a combination of electric and magnetic fields in which the electric field is radial in the xy plane and a constant magnetic field B lies along the z axis. The Hamiltonian for this system reads:

$$\hat{H} = \frac{1}{2M}p_i^2 + \frac{1}{2M}g\epsilon_{ij}p_i x_j + \frac{1}{2}M\omega^2 x_i^2 \tag{2}$$

where the coordinates x_i are in the laboratory frame, g is a constant, the two-dimensional antisymmetric unit tensor $\epsilon_{12} = -\epsilon_{21} = 1$, $\epsilon_{11} = \epsilon_{22} = 0$, and the mass M is the mass of the whole Rydberg atom, held on a Penning orbit with angular frequency ω by virtue of its dipole moment. Note that this dipole moment is a structureless constant in Zhang's paper, because the internal atomic structure has a negligible effect. In fact, we consider that Zhang's experiment is not realizable because of the magnitudes of the frequencies involved and because the radial electric field is a very difficult geometry to achieve.

In our proposal, the physical situation is completely different. The Hamiltonian of the Rydberg electron in crossed electric and magnetic fields has the general form:

$$\hat{H} = \frac{1}{2m}(p_i + eA_i)^2 + \Phi_c + \mathcal{F}x \tag{3}$$

where Φ_c is the coulomb field, $\mathcal{F}$ is the strength of a uniform external electric field acting in the x direction and A_i is the vector potential of the external magnetic field. We know that, for an appropriate choice of parameters, a minimum appears in the potential resulting from the combination of electric field, coulomb field and quadratic Zeeman terms (see Fig. 2 and Ref. [2]). It is therefore possible to choose a new set of coordinates (x_1', x_2', x_3') parallel to the (x, y, z) axes centred at this minimum and to perform a Taylor expansion around this new origin retaining only harmonic terms of the form $k_i x_i^2$ where all the k_i are positive constants. Note also that the Hamiltonian decomposes into a one-dimensional vibration along the x_3' axis with the simple harmonic form $\hat{H}_z = p_3^2/2m + m\omega_z^2 {x_3'}^2$ and and a two-dimensional motion in the $(x_1' - x_2')$ plane.

To represent the uniform magnetic field along x_3, we choose a gauge such that $A_i = B\epsilon_{ij}x_j/2$, where $(i, j) = (1, 2)$ and the normal summation convention is used. The first term involving A then reads $\frac{1}{2}\omega_L\epsilon_{ij}p_i x_j'$ where ω_L is the cyclotron frequency. This is simply the linear Zeeman term. The second (the quadratic Zeeman term) then reads $(m\omega_L^2)({x_1'}^2 + {x_2'}^2)/8$ and, since it is quadratic in x_i', can simply be subsumed into the harmonic terms of the Taylor expansion by adjusting the magnitudes of the constants k_i. Thus, the Hamiltonian of the $(x_1' - x_2')$ motion, in the Harmonic approximation close to the minimum of the outer well, becomes:

$$\hat{H} = \frac{1}{2m}{p_i}^2 + \frac{1}{2}\omega_L\epsilon_{ij}p_i x_j' + k_i {x_i'}^2 \tag{4}$$

which has the same form as Zhang's Hamiltonian in equation (2), except that m is the mass of the electron and that the k_i are different from each other, i.e. this is a cranked rather than a pure harmonic oscillator.

Since the algebraic form of this Hamiltonian is identical to that of Zhang, the mathematical structure of his original proof is completely preserved for the electron of a Rydberg atom in crossed electric and magnetic fields, close to the outer well of the double-well potential. This fact provides us with the opportunity for an experimental test under conditions more favourable than those envisaged by Zhang.

3 Experimental background

The phenomenon we propose as a test of String Theory is therefore the same as envisaged by Zhang, but in a more suitable system than the one he considered, for which the field geometry can be realized and the frequencies become accessible experimentally. Indeed, the systems to which we propose to apply Zhang's test can be made in the laboratory. We describe how this can be done, and explain what further work, both experimental and theoretical, needs to be accomplished in order to turn Zhang's suggestion into a viable experimental test. The atomic system we have identified as being experimentally suitable is a Rydberg atom in perpendicularly crossed external electric and magnetic fields. Such a system allows giant atomic dipoles to be created, and we propose to exploit a specific property of these dipole states in the parameter regime where a double-well potential occurs. The possibility of manufacturing huge electric dipoles was emphasized by Dzyaloshinskii [10], following classical orbit calculations by Clark et al. [11]. These authors showed that classical calculations predict a novel class of closed trajectory (see Fig. 1), in which the electron can orbit one or more times around the saddle point of the crossed-field potential. Since the electron can 'circle' a few times at a large distance to one side of the nucleus, the highly-excited atom in this peculiar quasi-bound state exhibits an anomalously large electric dipole moment.

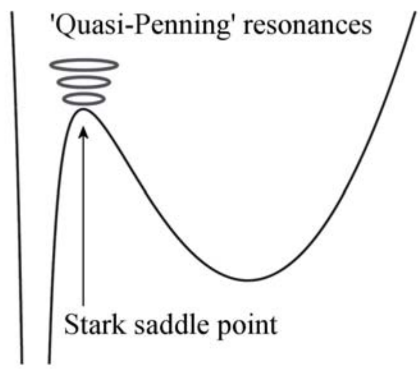

Fig. 1. The quasi-Penning orbits above the Stark saddle point for a hydrogenic atom in crossed electric and magnetic fields.

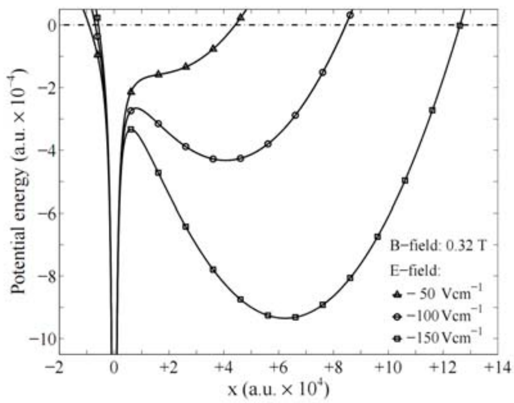

Fig. 2. The double-well potential in a strong magnetic field, as a function of electric field strength.

From the quantum perspective, the crossed-field potential has the remarkable feature of possessing a double well, in which the shape of the outer well can be controlled by varying the electric field strength (see Fig. 2). Double-well potentials producing hybrid states are well-known to produce non-Rydberg effects in highly-excited atoms [12], but the special feature of the ones considered here is that they are fully controllable using externally applied crossed fields.

Although such specially prepared double-well states have not been much studied theoretically, there must, in addition to the 'quasi-Penning' excitations just mentioned, exist states even further out from the nuclear centre, which are trapped in the outer well of the double-well potential, rather than 'surfing' over the saddle point. These states possess even larger dipole moments than the 'quasi-Penning' excitations. Evidence for the occurrence of 'quasi-Penning' states associated with the saddle point, or for the existence of bound states of the outer well of the double-well potential has not been obtained experimentally, but giant electric dipole states have been the subject of several theoretical studies. The most recent is that of Zöllner et al. [13], who have given a full quantum treatment, and who also include in their calculations many-electron effects. These effects were left out of the earlier classical calculations [11], all of which were based on a hydrogenic model. Thus, the double-well enhancement of giant electric dipole moments is found to be a general feature for many-electron atoms as well. The experimental status is less complete. Previous authors have searched for 'quasi-Penning' orbits in crossed fields under conditions for which a large electric dipole moment is expected in order to find evidence for the double-well structure [14], but to little avail, although some general arguments have been advanced in favour of their existence. We have carried out experiments, reported here, in crossed electric and magnetic fields for barium Rydberg atoms close to the $n \sim 37$ field-free state (a wavelength of 238.35 nm for a single photon transition from the ground state), using a novel geometry which allows full separation of the σ^- and σ^+ states. We propagate a collimated, effusive beam of barium atoms between the poles of a dipole electromagnet, perpendicular to the magnetic field direction. A pair of electric field plates are located at the centre of the magnet pole pieces such that an electric field can be applied in a direction mutually perpendicular to the magnetic field and the direction of propagation of the atomic beam. Barium Rydberg states are then excited via a single photon transition from the ground state by pulsed, single-mode, tunable laser radiation. As the laser radiation propagates through the crossed-field interaction volume, parallel to the magnetic field, the σ^- and σ^+ states can be selectively excited by choosing the sense of circular polarization of the radiation. Following interaction with the laser radiation, the atomic beam travels 55 mm downstream before the excited atoms are selectively field-ionised and the detached electrons are detected on a pair of micro-channel plates (MCPs). The combination of this novel experimental geometry with time-of-flight speed selection of the excited atoms upon detection allows the cancellation of the motional Stark effect to high precision, as described previously [15]. Thus the electric field experienced by the atoms as they traverse the crossed-field interaction volume may be completely controlled. The present experiments are a continuation of earlier studies in our laboratory under different crossed-field conditions [16,17].

Our observations of both linear and circular polarization spectra at fixed laser frequency while scanning the electric field reveal a strong modulation in the linearly polarized spectrum recorded (see Fig. 3). When considering this figure, note that a zero-level line has been included for each trace, and that each trace has its own scale indicated along the vertical axis. This observation is consistent with probing a dipole state, if the latter exists only in a restricted range of electric field strengths.

We have studied how the sharp structure within this modulation travels in energy as a function of magnetic field by scanning the electric field strength, and we have found that it follows the locus of the outer minimum of the applied double-well potential (see Fig. 4). Note that short lines are included to the left and to the right of each trace to indicate the zero level. A scale bar for the vertical axis is also inserted adjacent to the topmost trace. The important point in this figure is that there is a significant (and

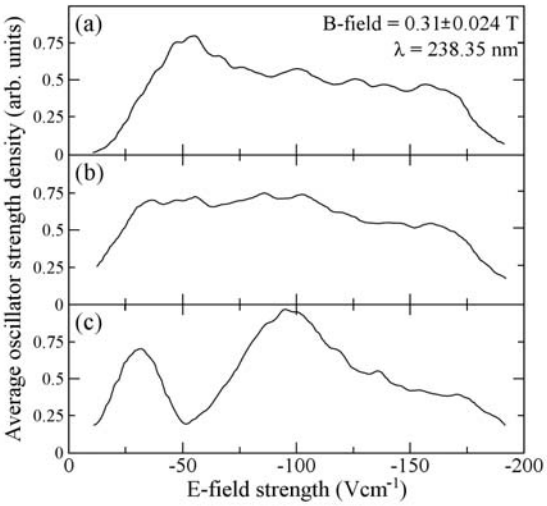

Fig. 3. Evolution of the absorption structure in crossed fields as a function of electric field strength for (a) light polarized linearly, parallel to the electric field axis, (b) circularly polarized light, and (c) light polarized linearly, perpendicular to the electric field axis. Note the pronounced modulation in (c).

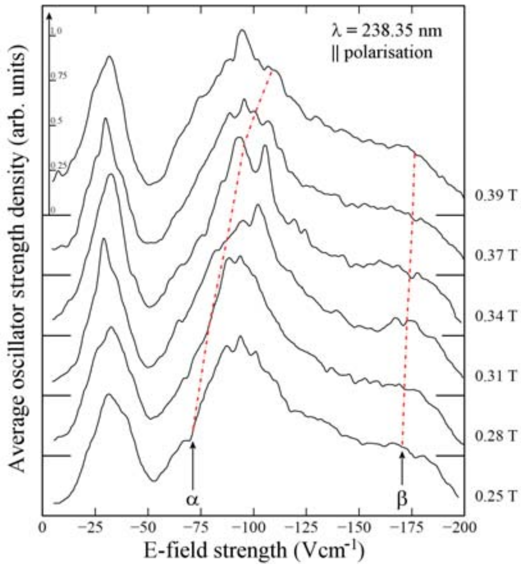

Fig. 4. Evolution of the absorption structure in crossed fields as a function of electric field strength for several different values of the magnetic field strength; α and β are, respectively, the electric field strengths for which the outer minimum in the double-well potential and the Stark saddle point cross over the fixed excitation energy.

reproducible) change in the fine structure as the parameters are varied, while the gross structure itself is stable and does not change significantly. In particular, consider the region in the centre of the figure along the line labeled α, where finer structure appears quite strongly at magnetic field strengths of 0.31 T, 0.34 T and 0.37 T. The dip close to the maximum at 0.34 T suggests the need for a finer scan of the electric field strength to gain further insight. This higher resolution electric field scan is presented in Figure 5.

Detailed scans in the range close to the Stark saddle point have also been performed to search for signs of the 'quasi-Penning orbits', but this has not proved fruitful. Indeed, we have found no experimental evidence at all for the existence of the predicted 'quasi-Penning' orbits. However, a high resolution scan of the range near the outer well minimum, achieved by holding the laser frequency fixed and fine-tuning the electric field strength has revealed spectral structure which we attribute to states bound in the outer well (see Fig. 5).

When considering the data of Figures 4 and 5, it is important to remember that varying the electric field strength alters the depth of the outer well. Thus, increasing the field strength produces new bound states which sweep down in energy, resulting in the structure which is displayed across the spectrum in Figure 5. Note that these states are also quasi-bound: their measured widths ($\sim$1 V cm^{-1}, corresponding to $\sim$10 kHz) are almost equal to the widths we estimate theoretically for tunneling between the outer and the inner well. Thus, two of the ingredients needed to achieve a test of String Theory along the lines of Zhang's theoretical principles are present in our experiment, namely (i) the excitation of a quasi-bound electron in crossed electric and magnetic fields and (ii) the manufacture of states of very high dipole moment. However, we do not propose to make use of Penning orbits, which anyway are not observed in our experiments.

Concerning Penning orbits above the saddle point and orbits in the outer well, we believe that, since Zhang's proof is based on a very general argument involving only commutation rules of angular momentum and position operators, it will actually turn out that either type of orbit in crossed fields would be suitable. Although the orbits of the outer well are not Penning orbits, they do share many of their properties. In particular, they evolve in a magnetic guiding field with a uniform crossed electric field applied. We have searched unsuccessfully for the predicted quasi-Penning orbits of the electron near the saddle point, but we have been successful in finding quasi-bound states of the outer well, and this is therefore the example we have concentrated on in Section 2 above.

What we lack to make further progress is a method to determine experimentally the orbital angular momentum in the lowest state of the outer well. Determining the angular momenta of several states in the outer well experimentally would therefore be a first step along the path initiated by Zhang.

A standard method of determining angular momenta experimentally is to use the linear Zeeman effect and to measure the g-factors of the states concerned. By analogy, a similar study of the Zeeman structure of the spectra in Figure 5 could be attempted, but requires a 'string theory of g-factors' to be developed, including the effects of spatial non-commutativity. The need for such a theory is one of the outcomes of the present study. Finally, there is the issue that Zhang's conclusions are strictly applicable in the zero-field limit. This poses problems, since the

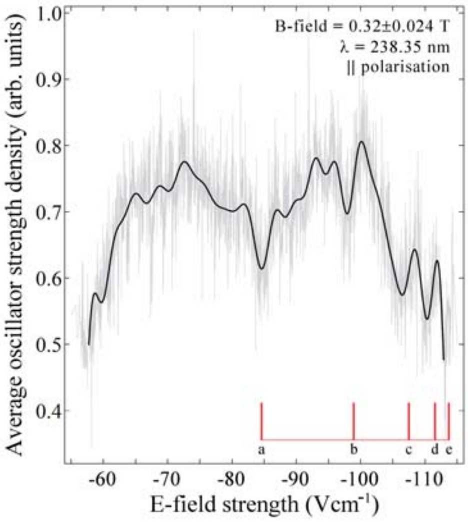

Fig. 5. Absorption structure in the range of the outer well (indicated by the lines labeled a to e), observed with the laser linearly polarized parallel to the electric field axis, by tuning the electric field strength with the laser wavelength fixed at 238.35 nm. Note that we have included the raw data (grey) in the background of this figure to emphasize that the structures are always present and do not emerge as a result of the 3-5-3 rolling averaging performed to obtain the solid black curve.

orbits (Penning, 'quasi-Penning' or outer-well) all become very large and consequently unstable in this limit. The answer, we suggest, is to measure g-factors as a function of field strengths and then extrapolate them down to the zero-field limit.

In conclusion, our experiments show that giant electric dipole states under conditions of crossed electric and magnetic fields are accessible experimentally for quasi-bound electrons in the particular geometry developed for our measurements. These states are of interest for an experimental test of String Theory, and suggest that Zhang's proposal could indeed be followed up, either in a more refined beam experiment along the lines we have described or by trapping and cooling single atoms under crossed-field conditions, so that motional Stark effects are eliminated. We further suggest that a 'string theory of g-factors' should be developed by taking spatial non-commutativity into account, so as to enable measurements of angular momentum to be performed.

The present experiment has been established with support from the Engineering and Physical Sciences Research Council (UK). We are grateful to the Royal Society of London and also to the Chinese Academy of Sciences for support under two International Joint Research Projects Nos Q739 and 16455. We are grateful to Dr. Crescencio Garcia-Segundo for his assistance in the experiments while at Imperial College, and for bringing some relevant publications to our notice.

References

1. Editorial, Nature Phys. **2**, 710 (2006)
2. P. Woit, *Not even wrong - The failure of string theory and the continuing challenge to unify the laws of physics* (Jonathan Cape, 2006)
3. L. Smolin, *The rise of string theory - the fall of science* (Houghton Mifflin, 2006)
4. J.-Z. Zhang, Phys. Lett. B **584**, 204 (2004)
5. J.-Z. Zhang, Phys. Rev. Lett. **93**, 043002 (2004)
6. J.-Z. Zhang, Phys. Lett. B **597**, 362 (2004)
7. Q.-J. Yin, J.-Z. Zhang, Phys. Lett. B **613**, 91 (2005)
8. J.-Z. Zhang, Phys. Lett. B **639**, 403 (2006)
9. J.-Z. Zhang, Phys. Rev. D, *Searching for Effects of Spatial Noncommutativity via Chern-Simons' Processes* (accepted for publication, 2007)
10. I. Dzyaloshinskii, Phys. Lett. A **165**, 69 (1992)
11. C. W. Clark, E. Korevaar, M.G. Littman, Phys. Rev. Lett. **54**, 320 (1985)
12. J.-P. Connerade, *Highly Excited Atoms* (Cambridge University Press, Cambridge, 2004)
13. S. Zöllner, H.-D. Meyer, P. Schmelcher, Europhys. Lett. **71**, 373 (2005)
14. G. Raithel, M. Fauth, H. Walther, Phys. Rev. A **44**, 1898 (1991)
15. R. J. Elliott, G. Droungas, J.-P. Connerade, J. Phys. B **28**, L537 (1995)
16. J.-P. Connerade, G. Droungas, N. E. Karapanagioti, M.-S. Zhan, J. Phys. B **30**, 2047 (1997)
17. J.-P. Connerade, M.-S. Zhan, J. Rao, K.T. Taylor, J. Phys. B **32**, 2351 (1999)